6th Symposium on Integrable Systems, Białystok 27-28.06.2013

Titles and abstracts

Mariusz Białecki

Instytut Geofizyki PAN, Warszawa

Title: *Random Domino Automaton and Combinatorial Explanation of the Shape of the Universal Curve of Earthquake Recurrence Time Distribution*

Abstract: This talk presents an explanation of a possible mechanism underlying the shape of the universal curve of Earthquake Recurrence Time Distribution. The presented simple stochastic cellular automaton model is reproducing the gamma distribution fit with the proper value of the parameter γ characterizing Earth's seismicity and also imitates a deviation from the fit at the short interevent times, as observed in real data.Thus the model suggests an explanation of the universal pattern of rescaled Earthquake Recurrence Time Distributions in terms of combinatorial rules for accumulation and abrupt release of seismic energy.

Maciej Błaszak

Uniwersytet Adama Mickiewicza, Wydział Fizyki, Poznań

Title: *Invariant quantizations of classical mechanics. Part I*

Abstract: In the lecture is presented an invariant quantization procedure of classical mechanics on the phase space over flat configuration space. Then, the passage to an operator representation of quantum mechanics in a Hilbert space over configuration space is derived. An explicit form of position and momentum operators as well as their appropriate ordering in arbitrary curvilinear coordinates is demonstrated. Finally, the extension of presented formalism onto non-flat case and related ambiguities of the process of quantization are discussed.

Jan Cieśliński

Uniwersytet w Białymstoku, Wydział Fizyki, Białystok

Title: *Improving the acuracy of the discrete gradient scheme*

Abstract: We present a class of nonstandard modifications of discrete gradient schemes for Hamiltonian systems of ordinary differential equations are proposed. Discrete gradient schemes preserve exactly (up to round-off errors) the energy integral. Our approach consists in replacing the time step h by some matrix function of h, keeping the conservative properties of the considered scheme and increasing its order.

Robert Conte

Centre de mathématiques et de leurs applications, École normale supérieure de Cachan, France.

Title: *Reduction of Gauss-Codazzi equations in three-dimensional Euclidean space to the sixth Painlevé equation*

(joint work with A. Michel Grundland, CRM, Montréal)

Abstract: Since the Gauss-Codazzi equations are underdetermined (three equations in four unknowns), we first restrict them to a determined system and compute its Lie point symmetries. This allows us to find a reduction to P6 more general than that of Hazzidakis (1897).

Adam Doliwa

Uniwersytet Warmińsko-Mazurski, Wydział Matematyki i Informatyki, Olsztyn

Title: *Non-commutative discrete KP hierarchy and its non-isospectral reductions*

Abstract: We investigate periodic reductions of Desargues maps, which lead to novel integrable multicomponent lattice systems being non-commutative, non-isosectral, and non-autonomous analogues of the modified Gel'fand–Dikii hierarchy. We show directly multidimensional consistency of the equations, and we present the corresponding systems of Lax pairs. We illustrate our approach on the lattice-modified Korteweg–de Vries and Boussinesq systems.

Alina Dobrogowska

Uniwersytet w Białymstoku, Instytut Matematyki, Białystok

Title: *Integrable systems of Neumann type*

Abstract: We construct families of integrable systems that interpolate between *N*-dimensional harmonic oscillators and Neumann systems. This is achieved by studying a family of integrable systems generated by the Casimir functions of the Lie algebra of real skew-symmetric matrices and a certain deformation thereof.

Ziemowit Domański

Uniwersytet Adama Mickiewicza, Wydział Fizyki, Poznań

Title: *Invariant quantizations of classical mechanics. Part II*

Abstract: In the lecture is presented an invariant quantization procedure of classical mechanics on the phase space over flat configuration space. Then, the passage to an operator representation of quantum mechanics in a Hilbert space over configuration space is derived. An explicit form of position and momentum operators as well as their appropriate ordering in arbitrary curvilinear coordinates is demonstrated. Finally, the extension of presented formalism onto non-flat case and related ambiguities of the process of quantization are discussed.

Piotr P. Goldstein

Narodowe Centrum Badań Jądrowych, Warszawa

Title: *Schrödinger equations with Tsallis entropy – not very new nonlinear equations*

Abstract: The classical Schrödinger equation (SE) may be derived in many ways. At least two of them may be generalized in a way compatible with Tsallis's approach to entropy. The first one (due to Tsallis et al.) relies on generalization of the exponential function, which thus generalizes the concept of the plane wave and consequently the corresponding SE. The other one (due to Olavo) starts from hydrodynamic equations and the Heisenberg relation between the position and momentum fluctuations. The latter may be expressed in terms of the entropy, yielding the classical SE for the traditional Boltzmann-Gibbs entropy and a generalized SE for the Tsallis entropy. The SE obtained by means of these two methods differ from each other. Both of the equations are nonlinear 2nd order PDE; only the 2nd one includes arbitrary potential, that equation is the subject of our analysis. Testing for the Painlevé property yields the Laurent expansion with negative Fuchsian indices; for physical reasons,! it seems that the chance for integrability is small. Physically relevant travelling-wave solutions will be derived and discussed.

Alfred Michel Grundland

Centre de Researches Mathematiques, Université de Montréal, Kanada

Title: Soliton surfaces via zero-curvature representation of integrable systems

(joint work with S. Post Hawaii University)

Abstract: A new version of the Fokas-Gel'fand formula for immersion of soliton surfaces in Lie algebras is presented. The teoretical considerations are illustrated via surfaces associated with Painlevé equations.

Andrzej J. Maciejewski

Uniwersytet Zielonogórski, Wydział Fizyki i Astronomii, Zielona Góra

Title: *Necessary conditions for integrability of algebraic Hamiltonian systems*

Abstract: Systems with Hamiltonian functions algebraic over the set of polynomials in coordinates and momenta are considered. For such systems the right hand sides of Hamilton equations typically are not single valued. This is why the Ziglin or the Morales-Ramis theory cannot be applied directly to study their integrability. One possible approach to solve this problem is proposed by Combot in T. Combot, "A note on algebraic potentials and Morales-Ramis theory". Celestial Mechanics and Dynamical Astronomy, 115(4):397-404, (2013). The main observation in this paper is following. One can extend the phase space of the system in such a way that the extended system is meromorphic and Hamiltonian with respect to a certain degenerated Poisson structure. Although the proposed construction is not unique it allows to obtain necessary conditions for the integrability in the framework of differential Galois theory. Applying this approach necessary conditions for integrability of algebraic homogeneous potentials of rational homogeneity degree are obtained. Presented results were obtained in collaboration with M. Przybylska

Michal Marvan

Silesian University, Institute of Mathematics, Opava, Czech Republic

Title: *On symmetries of the Gibbons-Tsarev equation*

Abstract: This is a report of a joint work in progress on the Gibbons-Tsarev equation. In particular, we present closed-form formulas for infinitely many nonlocal conservation laws and infinitely many nonlocal symmetries.

Maciej Nieszporski

Uniwersytet Warszawski, KMMF, Warszawa

Title: *Reductions of DIS. Discrete Painleve equations and Painleve correspondences*

Abstract: With many of difference equations one can associate a superior discrete integrable system closely related to Yang-Baxter maps. This association turns out to be helpful in understanding some correspondences (multivalued recurrences) that are consistent-around-the-cube. Moreover, the standard procedure of periodic reductions leads not only to Painleve equations but also to their multivalued generalizations.

Michał Olejniczak

Politechnika Poznańska, Poznań

Title: *Numerical meshfree method for KdV equation*

Abstract: In this talk I will give you a brief explanation what numerical meshfree methods are. I will show how one of them (Kansa method) can be applied with help of Picard technique to the KdV equation. Details of algorithm and numerical results will be discussed.

Anatoliy Prykarpatsky

Pedagogical State University, Drohobych, Ukraina.

Title: *The hidden symmetry analysis of Lax type integrable nonlinear dynamical systems within the Lie-algebraic, symplectic and differential-algebraic approaches*

Abstract: (see "Prykarpatsky" in Abstracts' submenu)

Maria Przybylska

Uniwersytet Zielonogórski, Wydział Fizyki i Astronomii, Zielona Góra

Title: *Quantization conditions in the Bargmann representation*

Abstract.: We formulate a universal method for solving an arbitrary quantum system which, in the Bargmann representation, is described by a system of linear equations with one independent variable, as N photon Rabi model, or N level system interacting with a single mode of the electromagnetic field.

We apply our method to the one photon Rabi model and its various generalisations. We prove that the spectrum of such models are just zeros of some transcendental functions. For standard Rabi system this transcendental function can be given e explicitly in terms of confluent Heun functions. Presented results were obtained in collaboration with A.J. Maciejewski and T. Stachowiak.

Stefan Rauch-Wojciechowski

Department of Mathematics, Linköping University, Sweden

Title: *Dynamics of inverting solutions of the tippe top*

Abstract: (see "Rauch-Wojciechowski" in Abstracts' submenu)

Alexander Sakhnovich

Vienna University, Austria

Title: *Generalized Darboux transformation: one and several variables*

Abstract: Generalized Bäcklund-Darboux transformation (GBDT) is a version of the well-known Bäcklund-Darboux transformation. The talk is dedicated to GBDT's applications to spectral theory and to the construction of explicit solutions of nonlinear equations. The case of several variables (e.g., nonstationary Schrodinger equation) is discussed as well.

Ekaterina Shemyakova

SUNY at New Paltz, New York, USA

Title: *Factorization of Darboux transformations of arbitrary order for 2D Schrödinger operator*

Abstract: We prove that a Darboux transformation of arbitrary order for 2D Schrödinger operator can be factored into Darboux transformations of order 1. Even for the special case of Darboux transformations of order 2 this problem is hard. For this case we have found earlier a rather beautiful proof based on the invariantization (we used regularized moving frames due to Olver and Pohjanpelto). The analogous statement for one-dimensional Schrödinger operator was proved in four steps (Shabat, Veselov and Bagrov, Samsonov). In this case the factorization is not unique, and different factorizations imply discrete symmetries related to the Yang-Baxter maps (Adler and Veselov). The paper is available at http://arxiv.org/abs/1304.7063

Tomasz Stachowiak

Uniwersytet Zielonogórski, Wydział Fizyki i Astronomii, Zielona Góra

Title: *Stokes phenomenon and quantization*

Abstract: In the Bargmann representation of a quantum system by entire functions, the derivation of spectra might follow both from analyticity and finiteness of the norm. In the latter, the key role is played by the asymptotic solutions at infinity and the Stokes phenomenon. The same approach can be used in the L^2 space which will be illustrated, for the Dirac equation, by means of the Laplace integral.

Błażej Szablikowski

Uniwersytet Adama Mickiewicza, Wydział Fizyki, Poznań

Title: *Integrable quantum Stäckel systems*

Joint work with M. Błaszak, Z. Domański and A. Sergyeyev.

Abstract: The Stäckel separability of a Hamiltonian system is well known to ensure existence of a complete set of Poisson commuting integrals of motion quadratic in the momenta. We consider a class of Stäckel separable systems where the entries of the Stäckel matrix are monomials in the separation variables. We show that the only systems in this class for which the integrals of motion arising from the Stäckel construction keep commuting after quantization are, up to natural equivalence transformations, the so-called Benenti systems. Moreover, it turns out that the latter are the only quantum separable systems in the class under study.

Adam Szereszewski

Uniwersytet Warszawski, IFT, Warszawa

Title: *Discretization of Projective-Minimal Surfaces*

(Joint work with W.K. Schief)

Abstract: The linear system for 2-dimensional real surfaces in 3-dimensional projective space will be discussed. The discrete analog of this system and its reduction to the system describing projective-minimal surfaces will be presented.

Wojciech Szumiński

Uniwersytet Zielonogórski, Wydział Fizyki i Astronomii, Zielona Góra

Title: *Dynamics of multiple pendula without gravity*

Abstract: I present the class of planar multiple pendula consisting of mathematical pendula and spring pendula in the absence of gravity. Among them are systems with fixed suspension point as well us unfixed ones. All these systems depend on 1parameters (masses, arms lengths) and possess circular symmetry S^1. I illustrate the complicated behaviour of their trajectories using Poincare section method. For some of them I prove their non-integrability on some hypersurfaces in space of parameters. Non-integrability results were obtained analysing properties of the differential Galois group of variational equations along certain particular solutions of the systems.

Irina Vereshchagina

Baltik State University, Kaliningrad, Russia

Title: *Projecting operators method development: propagation of waves in inhomogeneous medium*

Joint work with S. Leble.

Abstract: (see "Vereshchagina" in Abstracts' submenu)

Mikhail Vereshchagin

Baltik State University, Kaliningrad, Russia

Title: *Stability of relative equilibria in the unrestricted problem of a sphere and an axially symmetric body*

Abstract: My talk will be concerning such named the Full-two-body-problem (F2BP). It is different from the classical two-body problem (2BP). The difference is the following: unlike 2BP, in F2BP bodies are not spherical.and we can not neglect their shapes. This may be the case, for instance, when the bodies are close enough (for instance, binary systems). Unlike 2BP, F2BP in general case, is not integrable. Moreover, even for relatively simple shapes, such as ellipse, this problem is very complicated. I consider the particular case, of the F2BP, when one body is a sphere and the other is axially symmetric. In this case I have shown that there may exist three types of relative equilibria, regardless the explicit form of the potential of the non-spherical body. In my talk I will focus on stability investigation of those motions.