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Dynamics of inverting solutions of the tippe top

The tippe top has a shape of a truncated axially symmetric sphere with a peg attached to the flat surface. When spun sufficiently fast on its spherical bottom the tippe top turns upside down and continues motion on the peg.

Research on the tippe top has long history since 19-th century and it is presently understood that the gliding friction is responsible for inversion and that the inversion takes place for the values of parameters $1-\alpha < \gamma = I_1/I_3 < 1+\alpha$ where $0 < \alpha < 1$ measures the eccentricity of the center of mass and I_1, I_3 are the main moments of inertia.

The existing results say that all tippe top solutions approach asymptotically one of the asymptotic, frictionless solutions that are periodic global attractors. These results follow from the use of the LaSalle invariance priciple and analysis of stability of the asymptotic, frictionless solutions. They have existential character and they do not tell any further details about dynamical behavior of the symmetry axis $\hat{\mathbf{3}}(t)$ and other variables describing a dynamical state of the tippe top.

The central problem of analysing the dynamics of the tippe top remained unexplained. It requires analysis of strongly nonlinear, nonintegrable, dissipative dynamical system of **6** equation and it is indeed very difficult.

In recent years I have introduced a new method of studying dynamics of the tippe top that is based on deformation of integrals of motion of an integrable sub-case when the TT is rolling without sliding. This approach leads to one (nonintegrable) *Main Equation for the Tippe Top* that describes behavior of the inclination angle $\theta(t)$ between the symmetry axis $\hat{\mathbf{3}}$ and the vertical axis $\hat{\mathbf{z}}$.

Analysis of this equation, made together with Nils Rutstam, helped us to prove that during the inversion the axis $\hat{\mathbf{3}}(t)$ nutates fast within a narrow band that is moving from the nbhd of the north pole to the nbhd of the south pole of the unit sphere $\mathbf{S}^2$. It also allowed us to estimate the width of the nutational band and the frequency of nutations.

I shall demonstrate the motion of the tippe top and I shall present a computer simulation of solutions that confirm results of our analysis. They also show new subtle features of behavior generic solutions for an inverting tippe top.
