Projecting operators method development: propagation of waves in inhomogeneous medium

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We consider a generalization of the projecting operators method in the case of wave propagation in an inhomogeneous medium. We restrict ourselves by a one-dimensional evolution system of two equations with variable coefficients. $u_t(x,t) - a(x)Du - b(x)Dv = 0; \quad v_t(x,t) - c(x)Du - d(x)Dv = 0, \quad D = \frac{\partial}{\partial x}$. We specify the problem to cover the applications corresponding to the adiabatic acoustics [1]. Electrodynamics at the minimal one-dimensional level with fixed polarization is also reduced to the right and left waves in a hyperbolic problems [2, 3, 4]. We investigate the problem versions with a boundary regime and Cauchy problem constructing the projection operators for the equations that fix unidirectional modes. A method of successive approximations (a kind of perturbation theory) is developed via operator technique based on expansion of evolution operator as a function of the differential operators $\lambda(D) = \sum_n \lambda_n D^n$. Similar the case of constant coefficient two such operators $\lambda_\pm$ arise, the index marks the direction of wave propagation. The construction of the orthogonal projecting operators corresponds to this expansion, its form is a matrix one with non-abelian elements. The standard application of these projection operators allows to obtain evolution equations related to the separated directed waves. A nonlinearity and inhomogeneity are introduced as a right hand side perturbation which leads to generation of the the modes ot its interaction.

References

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